Using evidence accumulation modelling to quantify the relative contributions of spatial attention and saccade preparation in perceptual tasks.

Supplementary Materials
Experiment 1

Manifest analysis including absolute time bin

Accuracy.

The incremental model comparison procedure revealed that including the main effects of saccade congruency ($\chi^2(1) = 193.27, p < .001$) and cue validity ($\chi^2(1) = 39.07, p < .001$) both significantly improved the fit of the model, and therefore both terms were included in the final model. There was a main effect of saccade congruency. Accuracy was higher when the saccade target and discrimination target were congruent (target appeared at same location as saccade goal) ($M = 0.80, SD = 0.17, b = 0.82, SE = 0.06, z = 13.60, p < .001$), relative to when they were incongruent (target appeared at opposite location to saccade goal) ($M = 0.66, SD = 0.19$). The comparison procedure also revealed a slightly weaker main effect of cue validity. Accuracy was higher on trials in which the arrow was valid ($M = 0.76, SD = 0.17; b = 0.41, SE = 0.07, z = 6.32, p < .001$) relative to invalid ($M = 0.70, SD = 0.20$).

Reaction time.

The LMM comparison procedure revealed that the best model of RT was one that included a main effect of saccade congruency ($\chi^2(1) = 86.51, p < .001$), cue validity ($\chi^2(1) = 19.96, p < .001$), absolute time bin ($\chi^2(3) = 148.83, p < .001$) and a significant Saccade Congruency x Cue Validity interaction ($\chi^2(1) = 13.95, p < .001$). As is standard in reporting LMM results for RTs, a coefficient of magnitude at least twice its standard error (i.e. $|t| > 2$) was the criterion of significance. There was a main effect of bin. Compared to trials where saccade onset occurred within 100ms of target offset ($M = 770, SD = 204$), RTs were slower on trials where saccade onset occurred within 101-200ms of target offset ($M = 825, SD = 225, b = 68.97, SE = 6.85, t = 10.07$), 201-300ms ($M = 837, SD = 230, b = 89.43, SE = 8.87, t = 10.08$) and 301-450ms ($M = 840, SD = 201, b = 103.81, SE = 13.83, t = 7.51$). While the
main effects of saccade congruency and cue validity weren’t significant, there was a significant Saccade Congruency x Cue Validity interaction. The saccade congruency effect varied as a function of cue validity. We conducted two paired t-tests in order to determine the direction of the interaction. To correct for multiple comparisons our criterion of significance was a FDR adjusted $p$-value of .05. On trials in which the arrow was valid, RTs were significantly slower when the target and saccade goal were incongruent ($M = 839, SD = 144$) compared to congruent ($M = 779, SD = 107; t(23) = 2.90, p = .02$). When the arrow was invalid, there was no significant difference whether the saccade goal and target were congruent ($M = 831, SD = 171$) or incongruent ($M = 847, SD = 182; t(23) = 0.27, p = 0.80$).

**Saccade Latency.**

In order to examine the effect of saccade congruency and cue validity on saccade latencies, the LMM comparison procedure was conducted using saccade latency as the dependent variable. The LMM incremental modelling approach revealed the model of best fit was one including a three-way interaction between saccade congruency, cue validity and absolute time bin ($\chi^2(3) = 72.75, p < .001$). This interaction is best understood as a difference in the time it takes to initiate a saccade in the same or opposite direction to the cue. When arrow direction and saccade instruction conflict (make a saccade in the opposite direction to the arrow) saccade latencies are slower than when they do not (make a saccade in the same direction as the arrow), but only across the first two time bins. This interaction, however, is not evident across the last two time bins.
Supplementary Figure 1. Experiment 1 mean (A) proportion correct, (B) reaction time (ms) and (C) saccade latency (ms) as a function of saccade congruency, cue validity and absolute time bin. Error bars represent within subjects’ standard errors.
Linear Ballistic Accumulator Analysis

Model fits.

Figure 2 shows that the selected LBA model provides a reasonable account of the effects of saccade congruency, cue validity and SOA for dual task trials on average accuracy (Figure 2A) and central tendency (median) of RT for correct responses (Figure 2B). Note that evidence accumulation models are often assessed against their ability to fit the full distribution of RT as quantified by RT quantiles for faster and slower responses. Due to the relatively low numbers of responses in each cell of the design these estimates were highly variable and prevented us from being able to report them here. However, Figure 2C shows that the model does capture the scale of RT variability as quantified by plots of the standard deviations of correct RTs. Figure 3 similarly shows that the selected LBA model fit to discrimination-only trials also provided a reasonable account of the cue validity effect on both average accuracy (Figure 3A), central tendency of correct RT (Figure 3B) and standard deviation of RT on correct trials (Figure 3C).
A

![Graph A](image)

<table>
<thead>
<tr>
<th></th>
<th>Congruent</th>
<th>Congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invalid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Incongruent</th>
<th>Incongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invalid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B

![Graph B](image)

<table>
<thead>
<tr>
<th></th>
<th>Congruent</th>
<th>Congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invalid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Incongruent</th>
<th>Incongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invalid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correct RT (s) / 0.5 quantile

SOA (ms)
Supplementary Figure 2. Experiment 1 linear ballistic model fits to (A) error rates, (B) median correct RT and (C) correct RT standard deviations for dual-task trials. The left panels are the levels of saccade congruency when the cue is valid. The right panels are the level of saccade congruency when the cue is invalid.
Supplementary Figure 3. Experiment 1 LBA model fits to (A) error rates, (B) median correct RT and (C) correct RT standard deviations for discrimination-only trials. The left panels are the levels of saccade congruency when the cue is valid. The right panels are the level of saccade congruency when the cue is invalid.
Drift rate parameter estimates.

Dual-task trials. An LMM with rate estimates as the dependent variable revealed the model of best fit to include a main effect of SOA, $\chi^2(3) = 44.16, p < .001$, and the accumulator correspondence factor (C), $\chi^2(1) = 495.53, p < .001$. The quality of evidence accumulation can be indexed by the difference between the true and false accumulator (C). Larger differences represent a higher quality of signal from the target. The model of best fit included a significant Cue Validity x C interaction, $\chi^2(1) = 6.87, p = 0.009$. There was a larger difference between accumulators when the cue was valid relative to invalid (1.19 and 1.00). There was also a significant Saccade Congruency x C interaction, $\chi^2(1) = 101.64, p < .001$. A larger difference between accumulators occurred when the saccade goal and target location were congruent relative to incongruent (1.48 and 0.70). Importantly, the three-way interaction of Cue Validity x Saccade Congruency x C did not significantly improve the fit of the model, $\chi^2(1) = 0.06, p = 0.81$, suggesting that cue validity and saccade congruency both independently contributed to the quality of evidence accumulation.

In addition, we also used an LMM to examine effects on TRUE drift rate, which have their greatest influence on the overall speed of correct RT responses. The incremental modelling procedure confirmed that the model of best fit was one that included main effects of cue validity, $\chi^2(1) = 8.79, p = .003$, saccade congruency, $\chi^2(1) = 56.14, p < .001$, and SOA, $\chi^2(3) = 101.32, p < .001$, on the rate of the TRUE accumulator. Inspection of standard estimates revealed the drift rate of the TRUE accumulator to be faster when the arrow was valid (relative to invalid) ($b = 0.13, SE = 0.04, t = 3.43$) and the saccade goal and target location were congruent (relative to incongruent) ($b = 0.35, SE = 0.04, t = 9.08$). Compared to an SOA of 50ms the drift rate was higher on trials with an SOA of 100 ($b = 0.17, SE = 0.05, t = 3.02$), 150 ($b = 0.41, SE = 0.05, t = 7.52$) and 200ms ($b = 0.54, SE = 0.05, t = 9.83$).
**Discrimination-only trials.** The LMM modelling procedure with rate estimates as the dependent variable revealed a main effect of SOA, $\chi^2(3) = 19.40, p < .001$, and a main effect of accumulator (C), $\chi^2(1) = 275.63, p < .001$, to improve the fit of the model. A Cue Validity x C interaction, $\chi^2(1) = 67.26, p < .001$, also improved the fit of the model. Evidence accumulation was higher in quality when the arrow was valid relative to invalid (1.82 vs. 0.83).

An LMM with the drift rate of the TRUE accumulator as the dependent variable revealed the inclusion of a main effect of SOA only to improve the fit of the model, $\chi^2(3) = 8.89, p = 0.03$. Inspection of the standard estimates revealed a significant difference in drift rate between the 200ms SOA condition and the 50ms condition only ($b = 0.39, SE = 0.133, t = 2.90$).
LBA Modelling of Saccade Latency

**Bayesian sampling methods.**

For each model three times as many chains were used as model parameters. Sampling was carried out in two steps. First, sampling was carried out separately for individual participants in order to get reasonable start points for hierarchical sampling. The results of this step were then used as starting points for sampling the full hierarchical sample. During initial burn-in period there was a probability of .05 that a crossover step was replaced with a migration step. After burn in only crossover steps were used and sampling continued until the proportional scale reduction factor ($R''$) was less than 1.1 for all parameters, and also the multivariate version was less than 1.1 (Brooks & Gelman, 1998). Hierarchical estimation assumed independent normal population distributions for each model parameter. Population-mean start points were calculated from the mean of the individual-subject posterior medians and population standard deviation from their standard deviations, with each chain getting a slightly different random perturbation of these values. Hierarchical sampling used probability .05 migration steps at both levels of the hierarchy during burn in and only crossover steps thereafter with thinning set at 5 (i.e., only every 5th sample was kept), with sampling continuing until $R''$ for all parameters at all levels, and the multivariate $R''$ values, were all less than 1.1. The final set of chains were also inspected visually to confirm convergence.

**Priors.**

Priors were chosen to have little influence on estimation. Priors were normal distributions that were truncated below zero for $B$, $A$ and $sv$ parameters, and truncated at 0.1s for the $t0$ parameter (assuming that responses made in less than 0.1s are implausible). The $t0$ parameter was truncated above by 1s, and no posterior samples every approached this limit. There were no other truncations, so the $v$ prior was unbounded. The prior mean for $B$ was 1
and for \( A \) 0.5. The \( \nu \) parameter was given a prior mean of 1 and the \( s\nu \) parameter had a prior mean of 0.5. The \( t\theta \) parameter had a prior mean of 0.3s. All priors had a standard deviation of 2. Mean parameters of population distributions were assumed to have priors of the same form as for individual estimation, and the standard deviations of hyper parameters were assumed to have exponential distributions with a scale parameter of one. Plots of prior and posterior distributions revealed strong updating (i.e., posteriors dominated priors), making it clear that the prior assumptions had little influence on posterior estimates.

**Model fit.**

Figure 4 displays the fit of the LBA model to the saccade latency data in terms of defective cumulative distribution functions (lines) and 10\(^{th}\), 30\(^{th}\), 50\(^{th}\), 70\(^{th}\) and 90\(^{th}\) percentiles (points from left to right) averaged over participants. The left panel corresponds to when spatial cue and saccade location were congruent and the right panel corresponds to when they were incongruent. The thick black line and open points correspond to the data and the think grey lines solid black points to the model prediction averaged over posterior samples. The grey points correspond to percentile predictions for 100 randomly selected sets of posterior parameter samples, so their spread gives an idea of the uncertainty in the model’s predictions. Figure 4 shows that the average fit of the selected LBA model was excellent.
Supplementary Figure 4. Cumulative density function. Left panel shows the fit of the model to trials in which the saccade goal and the spatial cue were congruent. Right panel shows fit of the model for trials where the saccade goal and spatial cue were incongruent.
Experiment 2

Manifest analysis with absolute time bins.

Accuracy

As a first step we examined whether the saccade congruency or cue validity effects varied by visual angle from the target. An LME excluding congruent and valid trials revealed accuracy not to vary by degree of visual angle. That is, on incongruent trials, there was no evidence to suggest that accuracy differed whether a saccade was directed away from the target towards a placeholder positioned at a 90° angle or 180° angle. There was similarly no evidence to suggest that on invalid trials accuracy varied by whether the cue appeared at an angle of 90° from the target or 180° from the target. We therefore report the results of 180° trials only. The comparison procedure revealed that saccade congruency ($\chi^2(1) = 86.06, p < .001$) and cue validity ($\chi^2(1) = 60.01, p < .001$) both improved the fit of the model. Thus, both terms were included in the final model. There was a main effect of saccade congruency. Accuracy was higher when saccade goal and target location were congruent ($M = 0.92, SD = 0.14$) relative to when they were incongruent ($M = 0.83, SD = 0.22; b = -1.01, SE = 0.11, z = -9.46, p < .001$). The modelling approach also revealed a main effect of cue validity. Accuracy was best when the cue was valid ($M = 0.91, SD = 0.16$) relative to when the cue was invalid ($M = 0.83, SD = 0.22; b = -0.81, SE = 0.11, z = -7.62, p < .001$).

Reaction Time

Again, there was no evidence to suggest that for incongruent trials, RT varied as a function of whether the saccade target was directed away from the target towards a placeholder at an angle of 90° or 180°. Similarly, there was no evidence to suggest that RT varied whether the invalid cue was 90° from the target or 180° from the target. Therefore, we again report effects for 180° trials only. The LMM incremental modelling procedure revealed
that the inclusion of saccade congruency ($\chi^2(1) = 165.63, p < .001$) and cue validity ($\chi^2(1) = 43.98, p < .001$) both significantly improved the fit of the model. There was a main effect of saccade congruency. Responses were faster when the saccade goal and target location were congruent ($M = 787, SD = 182$) relative to when the saccade goal was opposite the target location ($M = 712, SD = 159; b = 94.18, SE = 6.93, t = 13.59$) (see figure 5B). Similarly, there was also a main effect of cue validity. Reaction times were faster when the cue was valid ($M = 739, SD = 175$) relative to when the cue was invalid ($M = 760, SD = 174; b = 46.02, SE = 6.92, t = 6.66$).

**Saccade Latency**

Unlike for accuracy and RT, saccade latency did vary by degree of visual angle. We therefore included all trials in our analysis of latency. When saccade latency was included as the dependent measure the LMM modelling approach revealed the model of best fit to be one that included a main effect of saccade congruency ($\chi^2(1) = 44.25, p < .001$), absolute time bin ($\chi^2(3) = 6759.10, p < .001$) and a Saccade Congruency x Cue Validity interaction ($\chi^2(1) = 55.26, p < .001$). Therefore, the final model included a main effect of saccade congruency, cue validity, time bin and a Saccade Congruency x Cue Validity interaction.
Supplementary Figure 5. Experiment 2 mean (A) proportion correct, (B) reaction time and (C) saccade latency as a function of saccade congruency, cue validity and absolute time bin.
Linear ballistic accumulator fits

Figure 6 demonstrates that the selected LBA model provides a reasonable account of saccade congruency and cue validity on average accuracy and the distribution of RT, which could be reliably assessed in Experiment 2 due to their being a larger number of trials per condition and participant. Figure 6A shows that the model provides a good account of error rate. Figure 6B represents the distribution of RT in terms of the 10th percentile (representing the fastest responses), 50th percentile (representing the average responses) and the 90th percentile (representing the slowest RT). The account given is quite accurate, with appreciable misfit only for the slowest RTs. This misfit may be the result of our task instructions emphasizing the accuracy of the task, rather speed, causing some re-checking of responses before responding on a minority of trials, a process that is not taken account of by the LBA model.
Supplementary Figure 6. Experiment 2 linear ballistic model fits to (A) error rates and (B) correct RT distribution (10th, 50th and 90th percentiles are lower, middle and upper lines respectively). The abscissa is cue validity and the panels are levels of saccade congruency.
LBA Modelling of Saccade Latency

Bayesian Sampling Methods

Model Fit. Figure 7 displays the fit of the LBA model to the saccade latency data in terms of defective cumulative distribution functions (lines) and 10th, 30th, 50th, 70th and 90th percentiles (points from left to right) averaged over participants. Moving left to right from the top left-hand panel. The first panel corresponds to trials in which the spatial cue and saccade location were congruent (excluding trails were a saccade was made to the upward placeholder). The next panel corresponds to trials where the spatial cue and saccade goal were congruent and in the placeholder above fixation. The next panel is trials in which the saccade goal and spatial cue were opposite. The lower left-hand panel are those trials in which the spatial goal was above fixation and the spatial cue was 90 degrees to the left or right of this location. While, the last panel was trials in which the saccade goal was above fixation and the spatial cue was at an angle of 90 degree to the left or right of this. The thick black line and open points correspond to the data and the think grey lines solid black points to the model prediction averaged over posterior samples. The grey points correspond to percentile predictions for 100 randomly selected sets of posterior parameter samples, so their spread gives an idea of the uncertainty in the model’s predictions. Figure 7 shows that the average fit of the selected LBA model was excellent.
Supplementary Figure 7. Cumulative density functions. From top left to bottom centre the panels show fit of the model to trials where the saccade goal and spatial cue were; congruent (LVF/RVF trials only); congruent and above fixation; directly opposite each other other; saccade goal was to the LVF/RVF and spatial cue was above fixation; and saccade goal was above fixation and spatial cue in LVF/RVF.
References